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A METHOD FOR SCALING THE HEAVE MOTION EQUATIONS OF THE C.A.B. 6-D.O.F. LOADS AND MOTION PROGRAM FROM MODEL TO FULL-SIZE CRAFT

Alex Gerba, Jr. and George J. Thaler

December 1977

Progress Report for Period Ending
September 1977

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A method has been developed for predicting the full-scale performance of the heave motion characteristics of the C.A.B. Surface Effect Ship under certain operating conditions. A simplified heave-only model is used to demonstrate the procedure of scaling model dimensions to full-sized craft and is validated using the 6-D.O.F. Loads and Motion Program for the 100-B craft. The results of scaling the heave-only model to a 3 K ton craft is

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SUMMARY

A method has been developed for predicting the full-scale performance of the heave motion characteristics of the C.A.B. Surface Effect Ship under certain operating conditions. A simplified heave-only model is used to demonstrate the procedure of scaling model dimensions to full-sized craft and is validated using the 6-D.O.F. Loads and Motion Program for the 100-B craft. The result of scaling the heave-only model to a 3 K ton craft is also presented.

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I. INTRODUCTION

Before construction is started on a new ship design, it is standard procedure to build and test models. Towing tank models supply specific data enabling the projection of small craft dynamic behavior to the larger craft through appropriate analytical and experimental techniques. In the case of the CAB-type Surface Effect Ship, there are special considerations that must be given to the bubble of air that provides the majority of the lift force and, therefore, strongly affects the heave motion dynamic characteristics.

In this report, a method has been developed for scaling the heave motion equations of the CAB, 6-DOF Loads and Motion Program, from model dimensions to full-sized craft and thereby obtain a prediction of the large craft heave motion characteristics. A simplified heave-only model (Reference 1) is used to demonstrate the procedure. The scaled equation results are validated to a good approximation using the Oceanics 6-DOF L & M Program for the 100-B craft. In addition, the results of further scaling to the 3 K ton craft is also presented.

II. DESCRIPTION OF THE HEAVE-ONLY MODEL

A simplified heave-only model of the XR-3 craft was developed and validated in Reference 1 for the purpose of obtaining a better understanding of the vertical motion characteristics of the CAB-SES. In this model, the pitch variations are reduced to zero by assuming 1) the center of pressure (CP) directly under the center of gravity (CG), 2) the sidewall symmetrical about the CG with uniform rectangular cross-section from bow to stern, and 3) the pitch moments of the seals and aerodynamics cancel each other. The lift force of plenum pressure and side-wall buoyancy oppose the craft weight. All other lift and drag forces are neglected since constant speed conditions are assumed.

In addition, it was assumed that the rear seal maintains a constant leakage area, that is, the seal follows the water level at a fixed separation.

Figure 1 shows a top and sideview of the simulated CAB craft. All dimensions are chosen to approximate those of the XR-3 craft and were used to compare the results of this analysis to the L & M program solution under similar operating conditions.

The equations of motion for this system are given below.

$$\text{Orifice Leakage Rate, } q_{out} = C_n A_\ell \sqrt{\frac{2\bar{P}_b}{\rho_a}} \quad \frac{\text{cu. ft.}}{\text{sec.}} \qquad (1)$$

$$\text{Fan Map Input Rate, } q_{in} = n\left[Q_i - k\bar{P}_b\right] \quad \frac{\text{cu. ft.}}{\text{sec.}} \qquad (2)$$

Absolute Plenum Pressure, (Adiabatic Process) $P_b = P_a\left(\frac{M_b}{V_b \rho_a}\right)^{\gamma}$ Psf. (3)

Plenum Volume, $V_b = (V_n - A_b \ell_d)$ cu. ft. (4)

Plenum Air Flow Rate, $\dot{M}_b = \rho_a(q_{in} - q_{out})$ $\frac{\text{slugs}}{\text{sec.}}$ (5)

Heave Acceleration, $\ddot{Z} = \left(\frac{W}{M} - \frac{F_p}{M} - \frac{F_\ell}{M}\right)$ $\frac{\text{ft.}}{\text{sec.}^2}$ (6)

Heave Velocity, $\dot{Z} = \left[\int \ddot{Z}\, dt\right]$ $\frac{\text{ft.}}{\text{sec.}}$ (7)

Heave, $Z = \left[\int_0^{\tau} \dot{Z}\, dt + Z(o)\right]$ ft. (8)

Plenum Pressure Lift Force, $F_p = A_b \bar{P}_b$ lbs. (9)

Buoyancy Lift Force, $F_\ell = \left[2(A_s \ell_d)\rho g\right]$ lbs. (10)

Plenum Gage Pressure, $\bar{P}_b = (P_b - P_a)$ Psf. (11)

Draft, $\ell_d = Z + Zs$ feet (12)

The system parameters and constants are listed below.

Adiabatic process coefficient,	$\gamma = 1.4$
Leakage area,	$A_\ell = Y_L\ Z_L = 2.50$
Leakage orifice coefficient,	$C_n = 0.90$
Air density,	$\rho_a = .002378$
Atmospheric pressure,	$P_a = 2116.$
Plenum area,	$A_b = 200.$

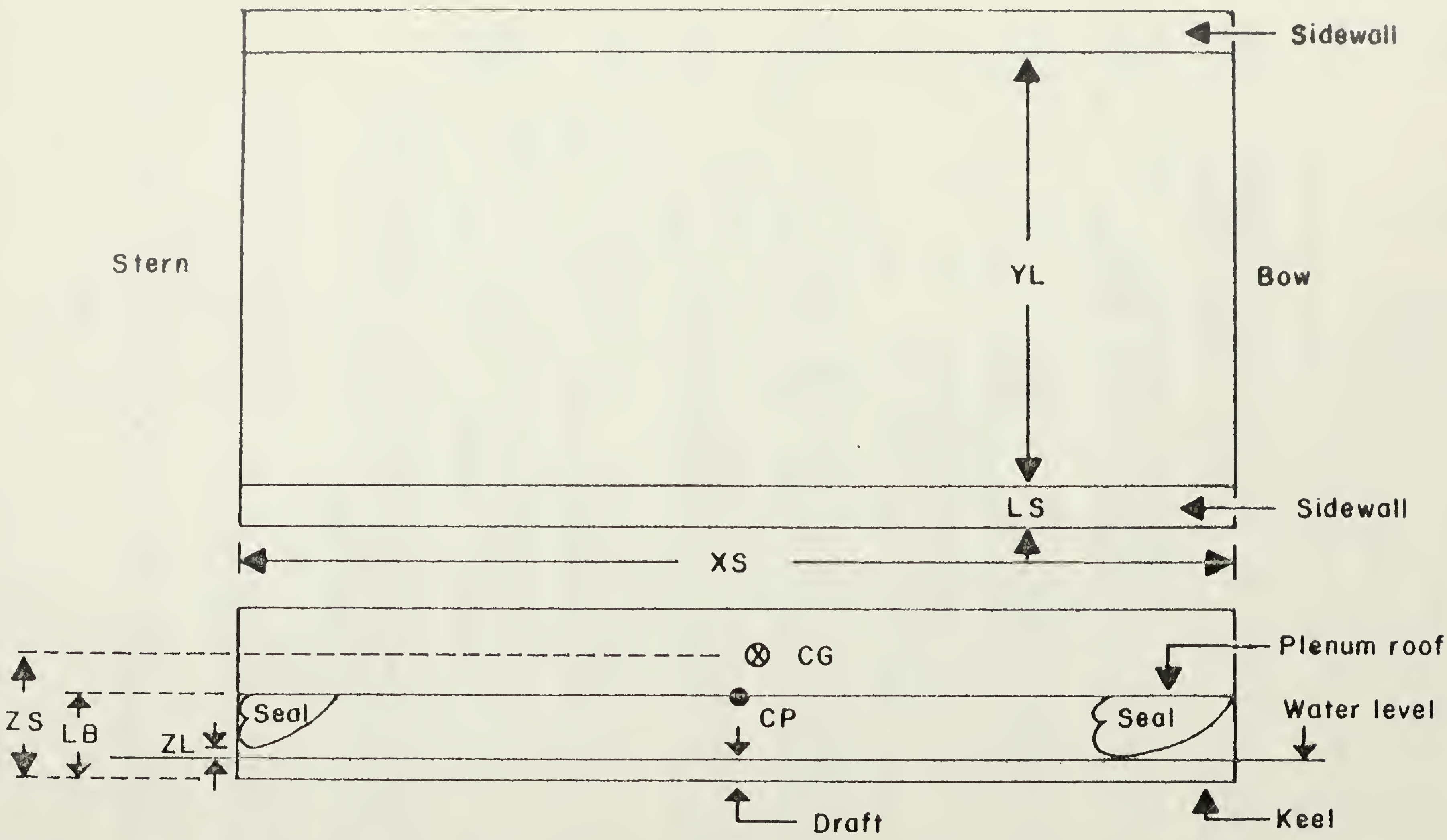


Figure 1.—Top and Side View of Simplified XR-3

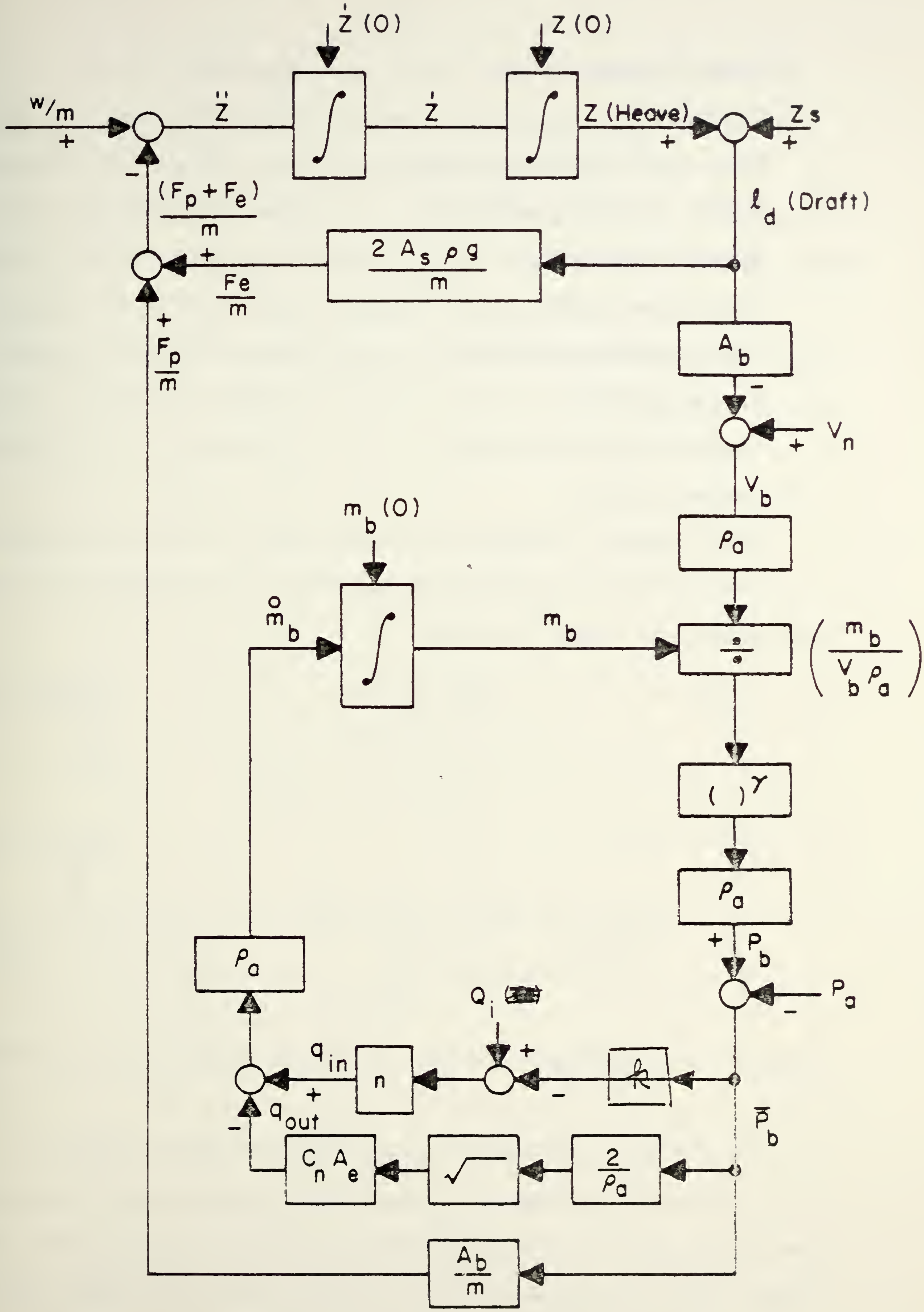


Figure 2.—Schematic Diagram of Signal Flow.

Empty plenum volume,	$V_n = 383.$
Craft weight,	$W = 6720.$
Keel line area of sidewall,	$A_s = 75/4$
Draft, initial condition,	$\ell_d(o) = 0.36$
Density of water,	$\rho = 1.99$
Gravity acceleration,	$g = 32.17$
Center gravity location,	$Zs = 2.5$
Number of fans,	$n = 8.$
Steady state fan output,	$Q_i = 64.3$
Fan map slope,	$k = 0.693$

The schematic diagram of signal flow is shown in Figure 2.

The coordinate system and equilibrium conditions for heave and draft are shown in Figure 3.

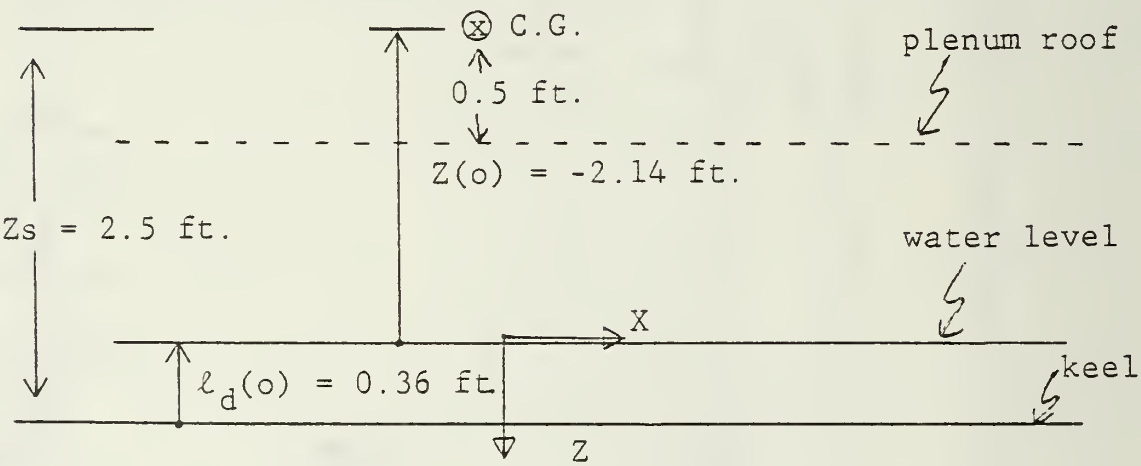


Figure 3. Coordinate and System and Initial Condition of Heave and Draft.

It should be noted that the range of heave motion is $-2.5 < Z < -0.5$ since -2.5 feet would put the water level at the keel line and -0.5 feet would put the water level at the plenum roof. Draft is measured from the keel and would have the range

$0 < \ell_d < 2.0$. Also note that if the craft "drops" into the water, ℓ_d increases and Z becomes less negative, that is, Z increases in the positive (downward) direction.

At steady-state condition, the plenum pressure assumed is $\bar{P}_b(o) = 29.27$ PSF producing a $F_p(o) = 5854$ lbs. For the draft of $\ell_d(o) = 0.36$ feet, the buoyancy force $F_\ell(o) = 866$ lbs.

The air mass in the plenum for the above steady-state was $M_b(o) = 0.612359$ slugs and the air flow rate was $q_{in}(o) = 353.$ cu.ft./sec.

III. SCALING METHOD WITH 6-D.O.F. MODEL VALIDATION

The scaling method used in this report has the underlying assumption that the mass density of the model and the full-size craft are the same and that the two ships are geometrically similar. Speed dependent effects such as frictional resistance are not considered in this model but again the assumption is that the ships are operated at corresponding speeds as given by Froude (Reference 2).

The idea then is to scale the model linear dimensions by the scale factor, $\lambda = L_{fs}/L_m$ where L_{fs} is the full-scale ship linear dimensions and L_m is the model linear dimensions. Thus, the full scale craft linear dimensions, including draft, are obtained by multiplying model dimension by the scale factor λ. All model areas, including the leakage area, would be increased by λ^2. The volumes and the weight of the model would be increased by λ^3.

The scaling of ambient pressure is not required since the effects of ambient pressure on the vertical plane motion have bee shown in Reference 4 to have negligible effects on both the pitch and heave motion at standard atmospheric pressure. It is importa to note that in the heave motion equations, the plenum pressure is determined by the adiabatic process, that is, $P_b = P_a(M_b/V_b\rho_a)$ However, for a given model operating at a specific draft, the required plenum pressure, $\overline{P}_b$, is obtained by subtracting the required buoyancy force from the weight and dividing the result by the cross-sectional area of the plenum. Once this value of

$\bar{P}_b = (P_b - P_a)$ is obtained, it is necessary (as stated in Reference 1) to initialize the air mass, M_b in the computer program, in order to obtain a balance of the adiabatic process equation at the desired operating point.

In addition to the geometrical scaling of the model dimensions, it was also found necessary to apply the scale factor to the slope of the fan map curve. In order to obtain similar ship characteristics, the slope of the fan map curve at the operating point must be scaled by the square of the scale factor λ. The reason for using λ^2 can be obtained from the linear model characteristic equation developed in Reference 1 and included in this report in Appendix B. The linear model characteristic equation is third order; but as shown in Reference 1 and in Appendix B, the second order approximation $s^2 - a_{33}s - a_{21} = 0$ yields good results for the step weight transient response. The damping coefficient $-a_{33}$ is directly related to the sum of two terms where one term is the fan map slope, k_q, and the other term is the leakage area, A_ℓ. Therefore, to get similar ship characteristics, the same procedure used for scaling area (λ^2) must be used on the fan map slope.

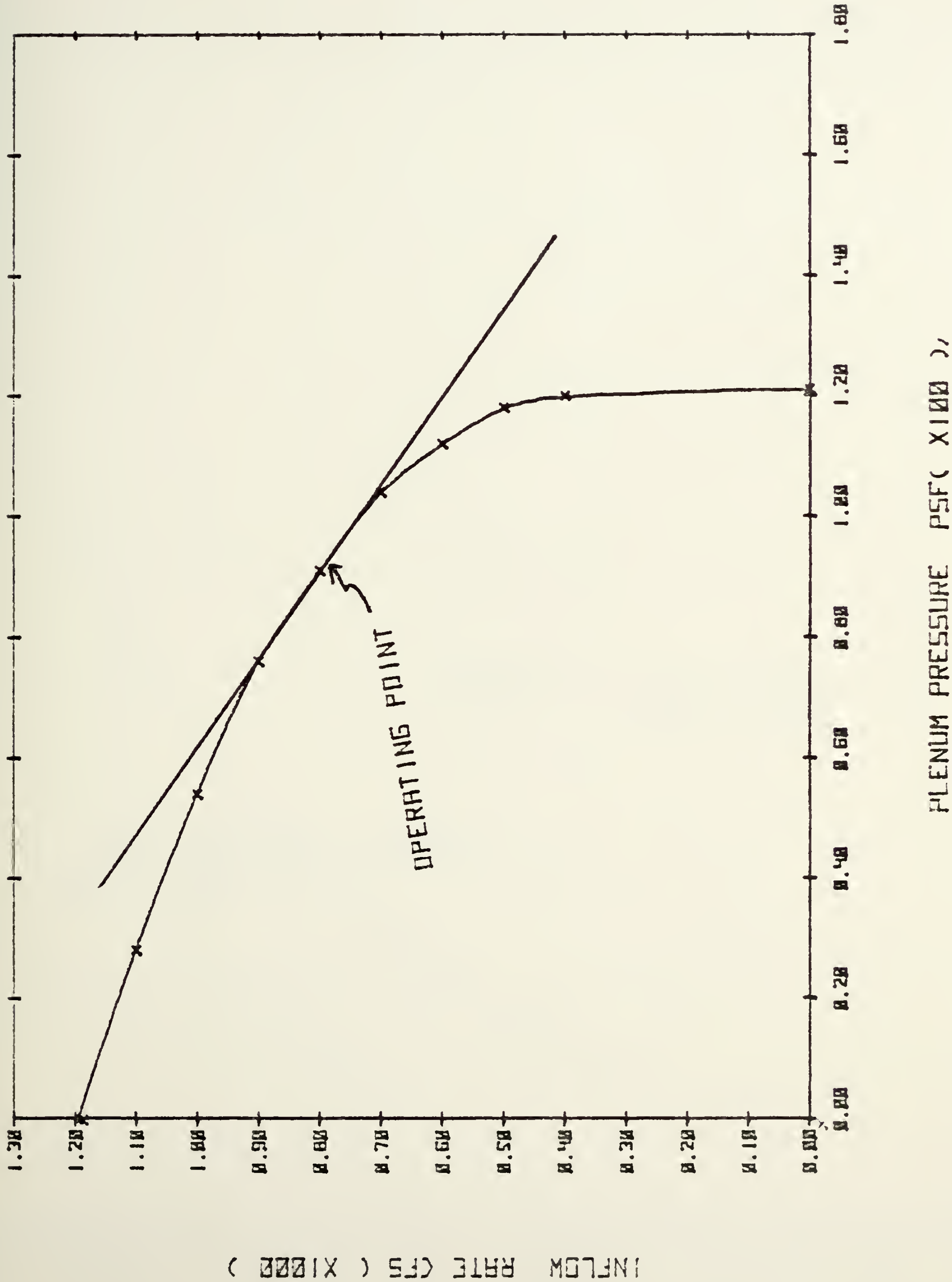
Equations 1 through 12 were programmed on the IBM 360/67 using the IBM program, "Digital Simulation Language," (DSL). The complete listing of the programmed equations is given in Appendix A, where SF represents the scale factor, λ. It should be noted that the leakage area used for the three-ton model in this report does not represent the XR-3 model presently in use at the Naval Postgraduate School (Reference 3). It was necessary to increase the leakage area to a large value in order to validate the scalin method by direct comparison with the Oceanic's 6-D.O.F. model of the 100-B craft.

Using a scale factor of 3.15, the three-ton craft was geometrically scaled to the 100-ton size craft. As stated earlier, an important consideration in obtaining similar ship response in heave motion is the slope of the fan map characteristics. It is necessary to change the slope of the fan characteristic curve by the square of the scale factor λ in order to obtain the desired heave response. Figure 4 shows the bubble fan map of the 100-B craft. The straight line drawn on the curve is the fan characteristic of the 100-ton craft obtained by scaling the three-ton fan slope with a scale factor $\lambda^2 = (3.15)^2$.

The validation procedure used in this report was a 10 percent step weight transient. This type of disturbance was also used to compare the results of the scaling procedure as given in the next section.

The response of the 100-ton craft is shown in Figure 5 and the 100-B craft in Figure 6. Table I shows a comparison of the two craft for the same step weight disturbance. It can be seen from the data that the CG acceleration and increment of draft change for the five second transient are within reasonable agreement.

FIG.4 AIR FLOW RATE VS PLENUM PRESSURE



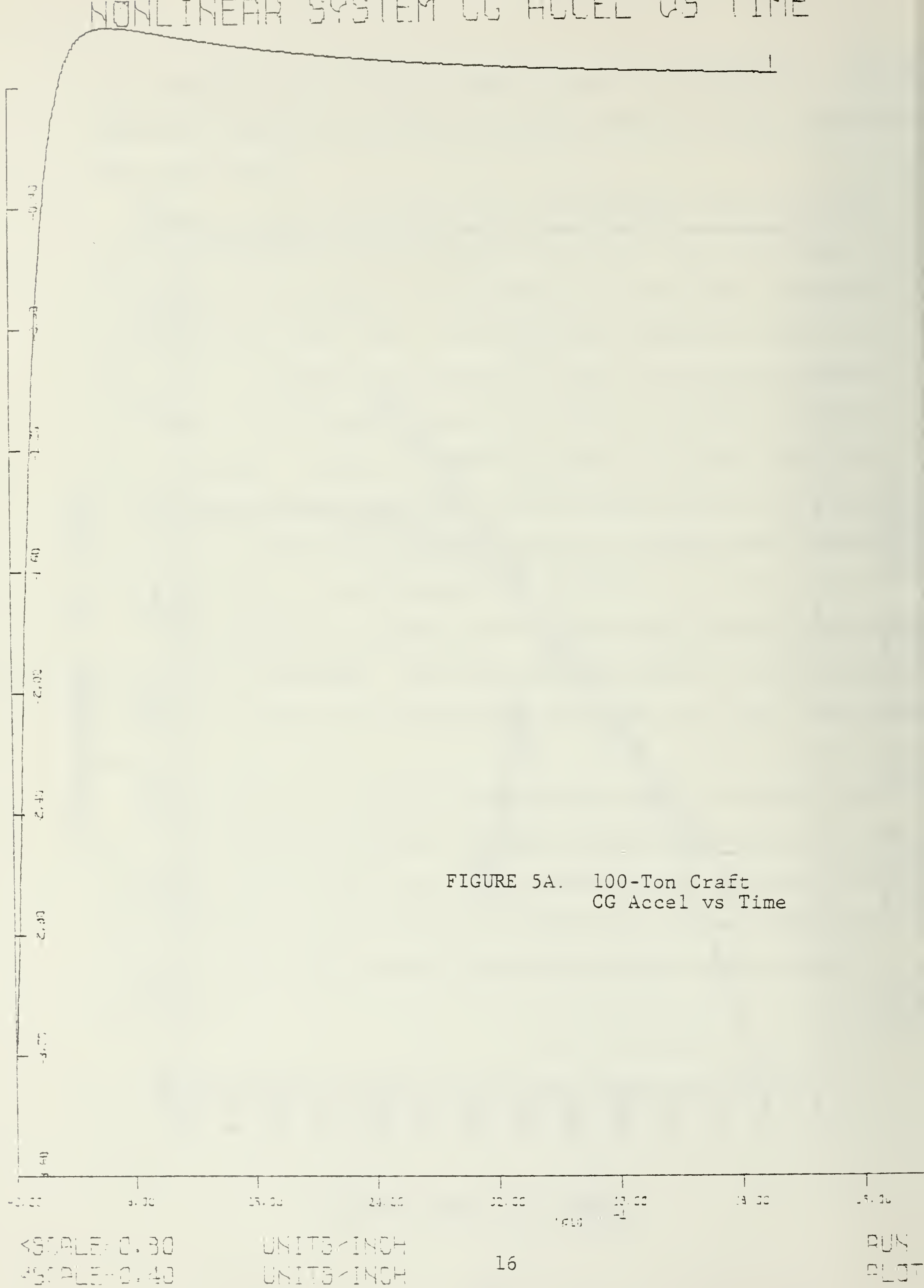


FIGURE 5A. 100-Ton Craft
CG Accel vs Time

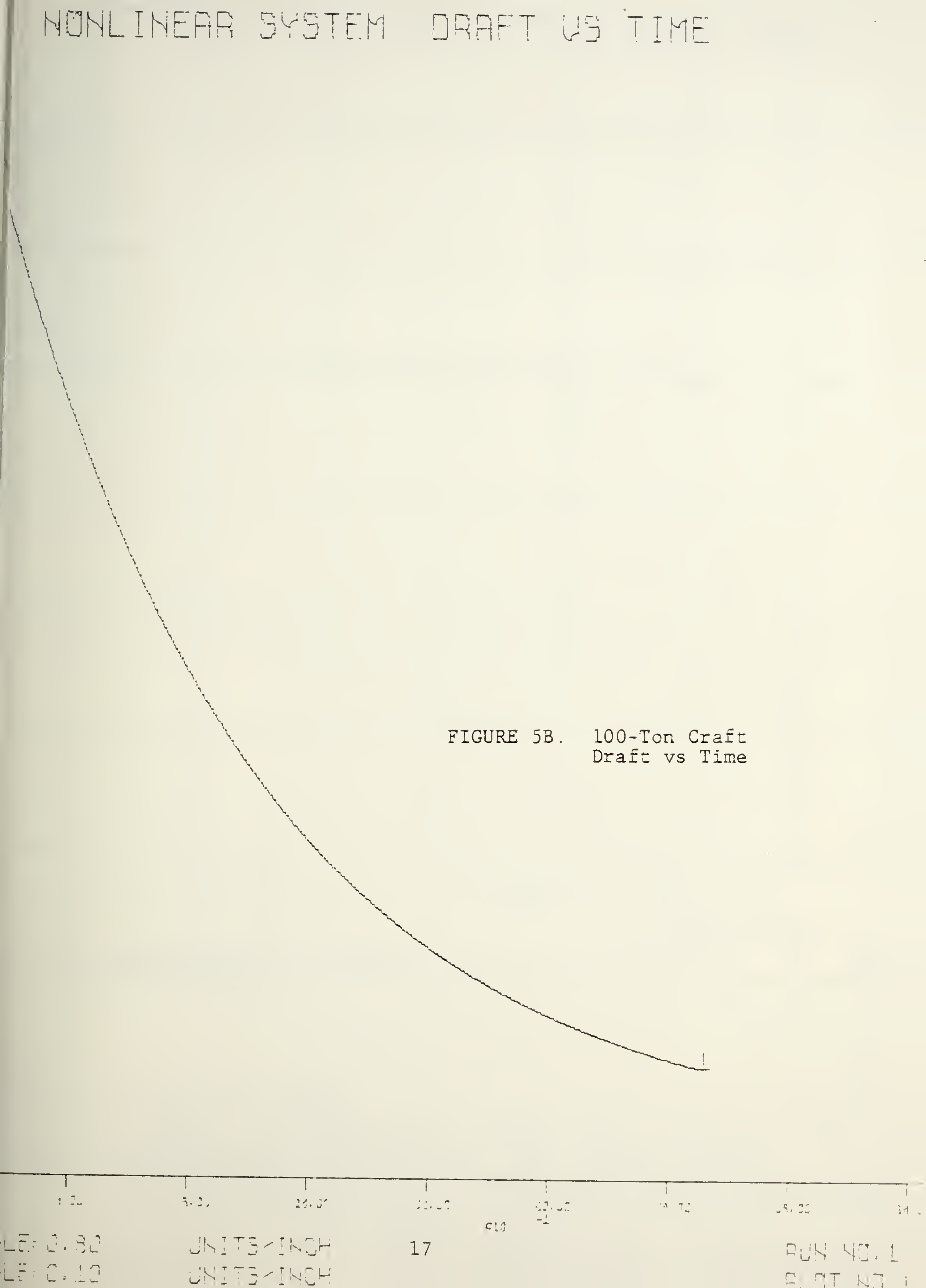


FIGURE 5B. 100-Ton Craft
Draft vs Time

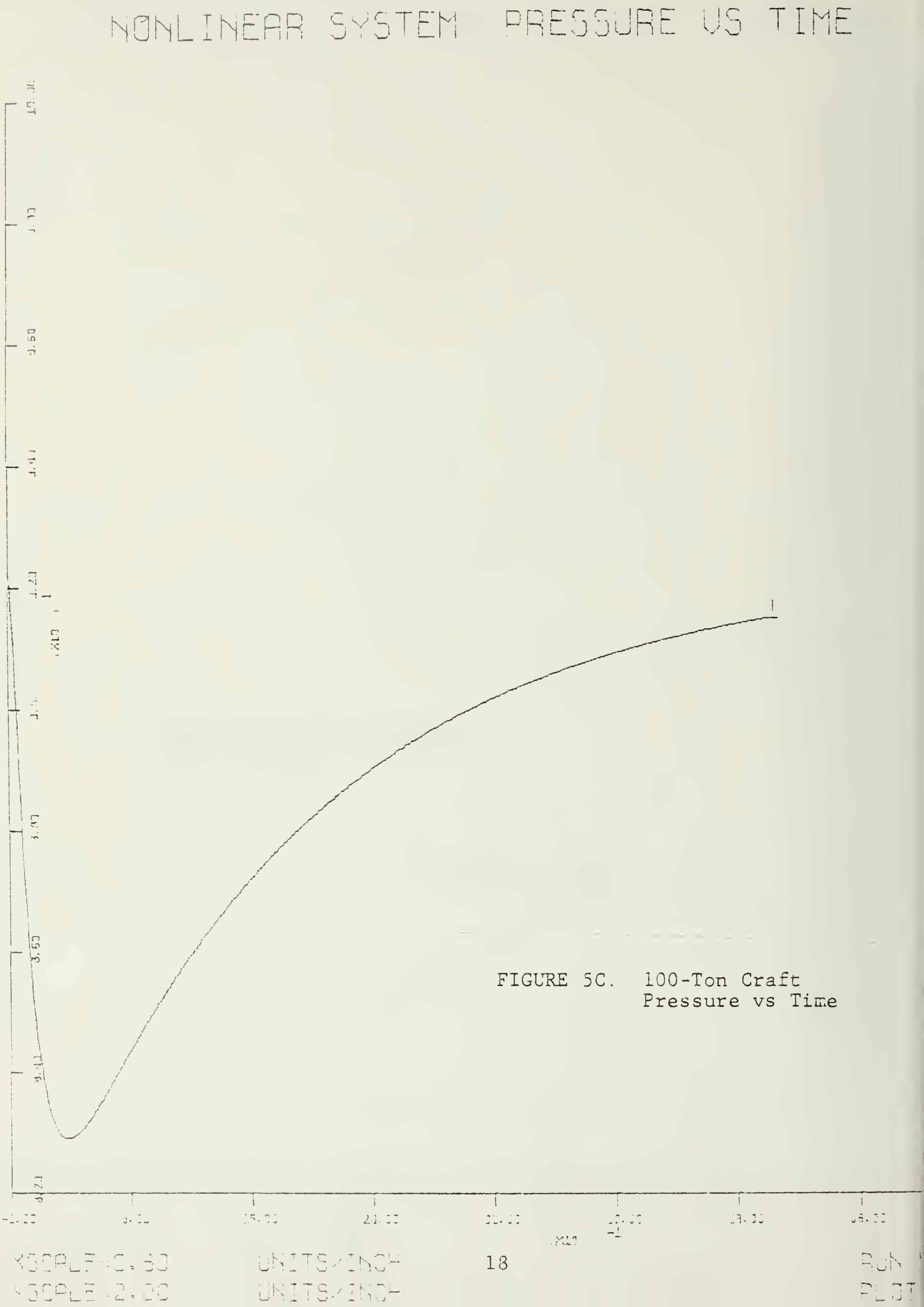


FIGURE 5C. 100-Ton Craft
Pressure vs Time

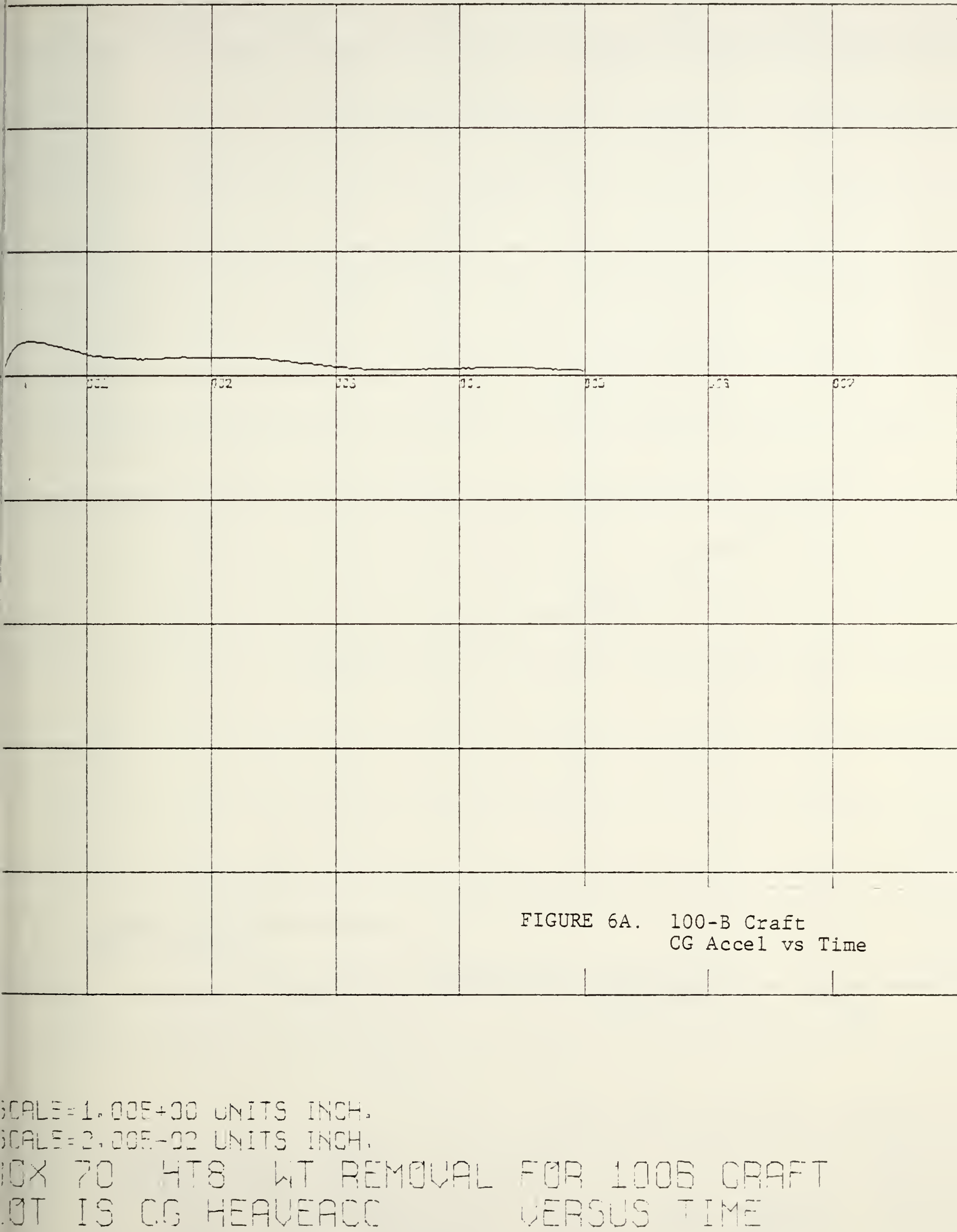


FIGURE 6A. 100-B Craft
CG Accel vs Time

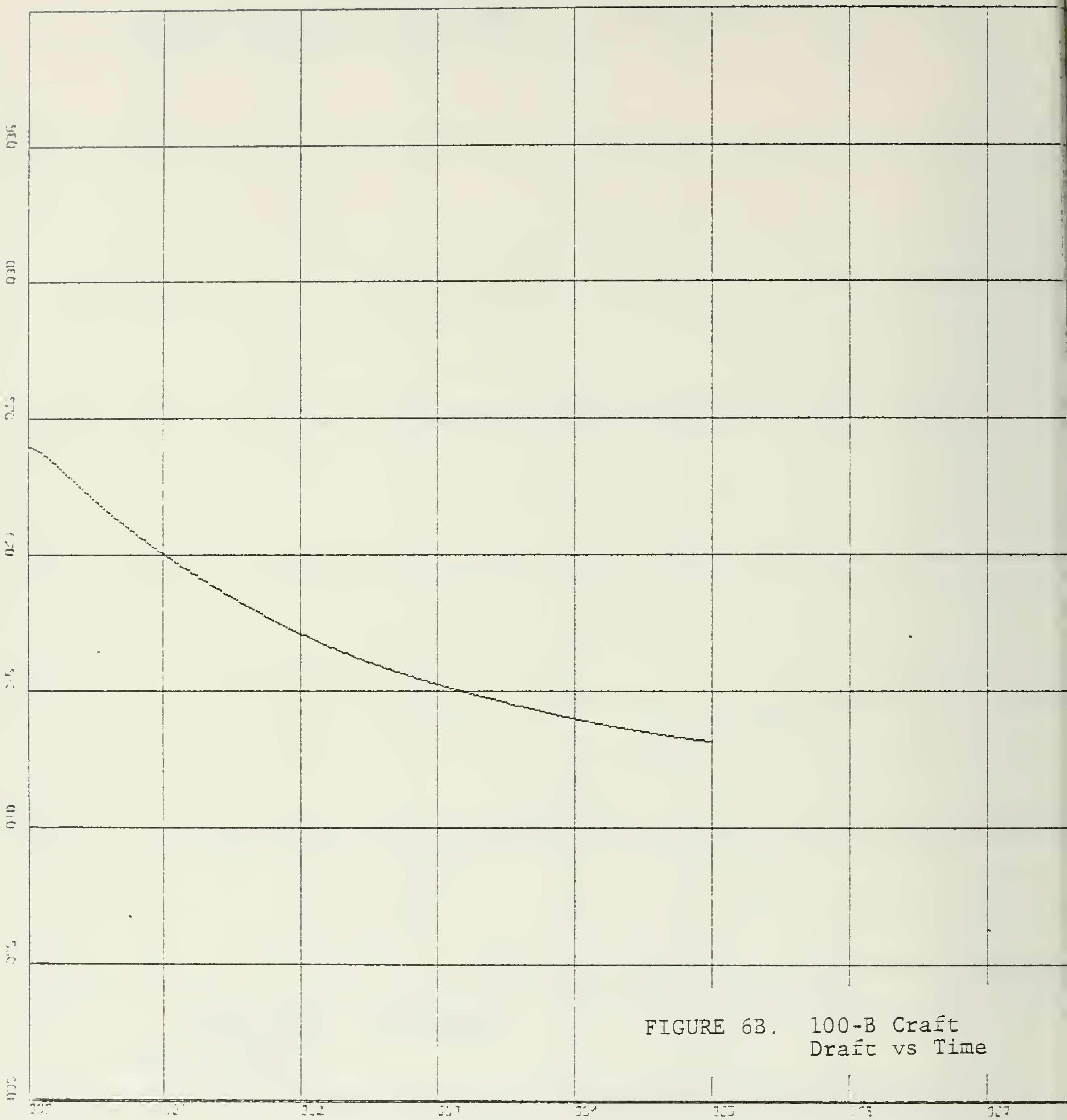


FIGURE 6B. 100-B Craft
Draft vs Time

X-SCALE=1.00E+00 UNITS INCH.
Y-SCALE 5.00E+00 UNITS INCH.
BOX 70 FTS WT REMOVAL FOR 100B CRAFT
PLOT IS DRAFT VERSUS TIME

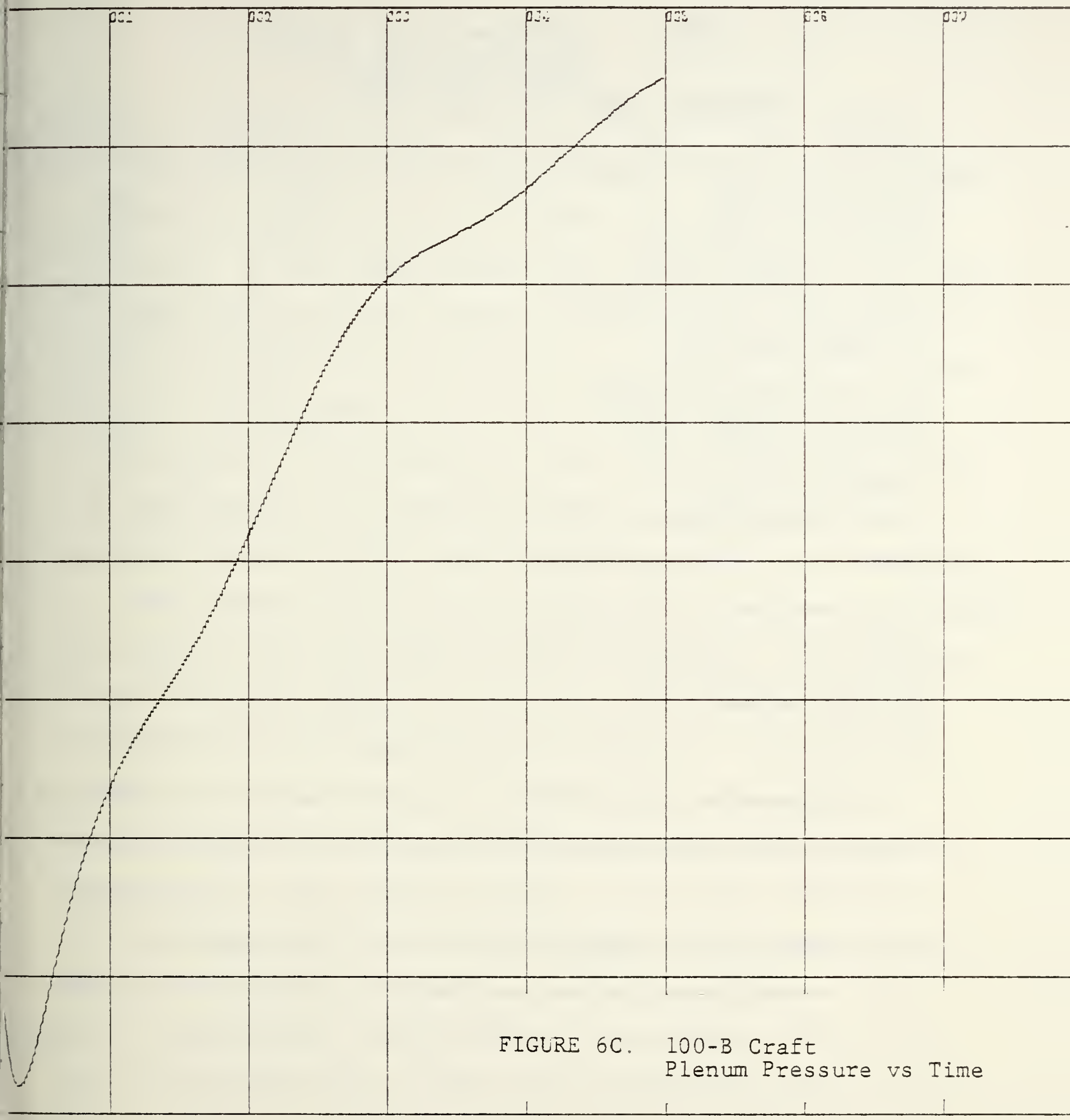


FIGURE 6C. 100-B Craft
Plenum Pressure vs Time

SCALE=1.00E+00 UNITS INCH.
SCALE=1.00E+00 UNITS INCH.
30X 70 HTS WT REMOVAL FOR 100B CRAFT
LOT IS PLENUM PRESSURE VERSUS TIME

TABLE I. Verification of Scaling Procedure Using Step Weight Disturbance

CRAFT		100-TON	100-B
Weight (K lbs)		210	210
Cushion length (ft)		65.31	65.31
Scale factor, $\lambda = L_{fs}/L_m$		3.15	3.15
Leakage area (ft^2)		24.80	25.52
Fan map slope (CFS/PSF)		6.88	6.88
Plenum pressure (PSF)		92.2	92.3
Number of fans, n		8	8
C.G.	t_p (sec)	0.675	0.51
Acceleration	% overshoot	6.10	5.1
Draft increment, (ft) (0-5) second		0.82	0.90

It should be pointed out that in Reference 1, it was shown that the simplified heave-only model could be made to closely approximate the 6-D.O.F. equation results for the step weight disturbances even though the effects of pitch motion on the total leakage area are not considered in the simplified model. Due to time constraints, no great effort was expended in obtaining a closer agreement in the 100-ton craft response for the above reason.

IV. STEP WEIGHT TRANSIENT RESPONSE

The use of a step-type disturbance as a rapid check on the transient characteristics of a system is a well established practice. A step weight removal was chosen because of its ease of application and also because of its possible utilization as a practical method for the validation of the XR-3 6-D.O.F. equations. The procedure is to start with the system in steady-state operation at the heavier weight and at the initial time of the transient, a 10 percent weight is removed and the response calculated for the first five seconds of the transient.

The response of a three-ton craft with dimensions approximate to those of the XR-3 craft is shown in Figure 7. The three-ton craft was then scaled (λ = 3.15) to a 100-ton size with the transient response shown earlier in Figure 5. The three-ton craft was further scaled (λ = 9.63) to the dimensions of a 3K-ton craft and had the response characteristics as shown in Figure 8.

A comparison of the characteristics of the three-ton, 100-ton and the 3000-ton ships is shown in Table II.

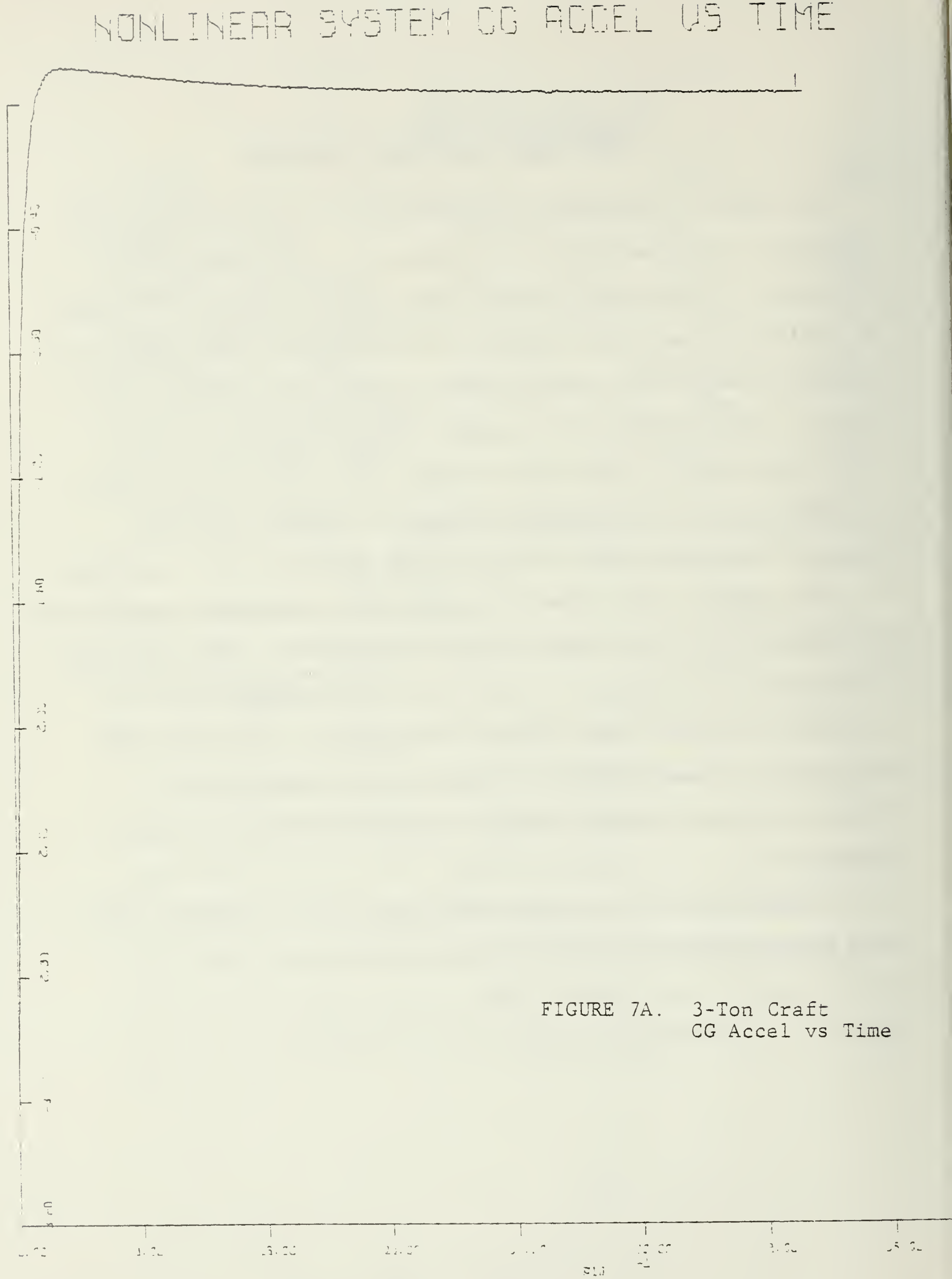


FIGURE 7A. 3-Ton Craft CG Accel vs Time

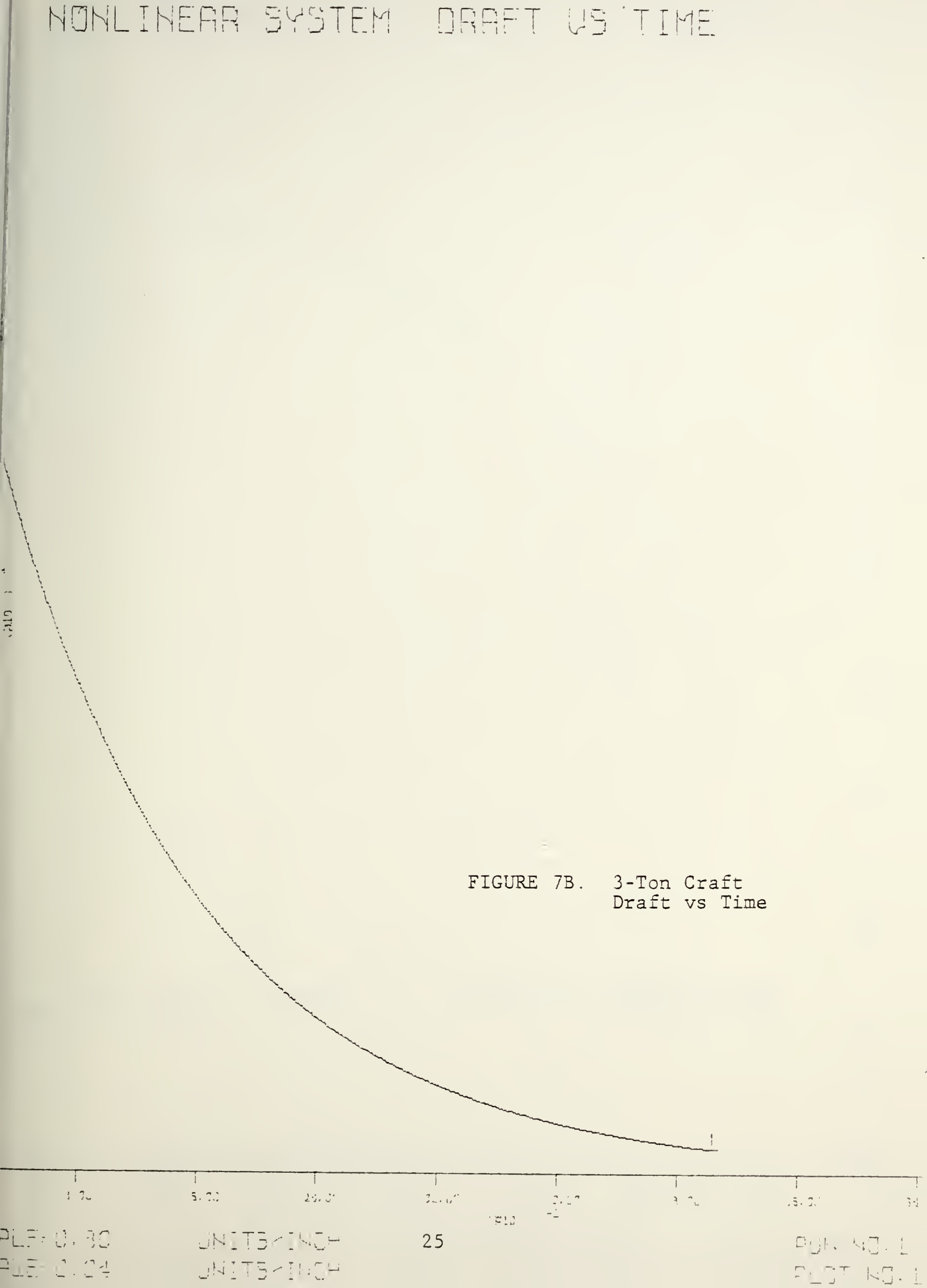


FIGURE 7B. 3-Ton Craft
Draft vs Time

NONLINEAR SYSTEM PRESSURE VS TIME

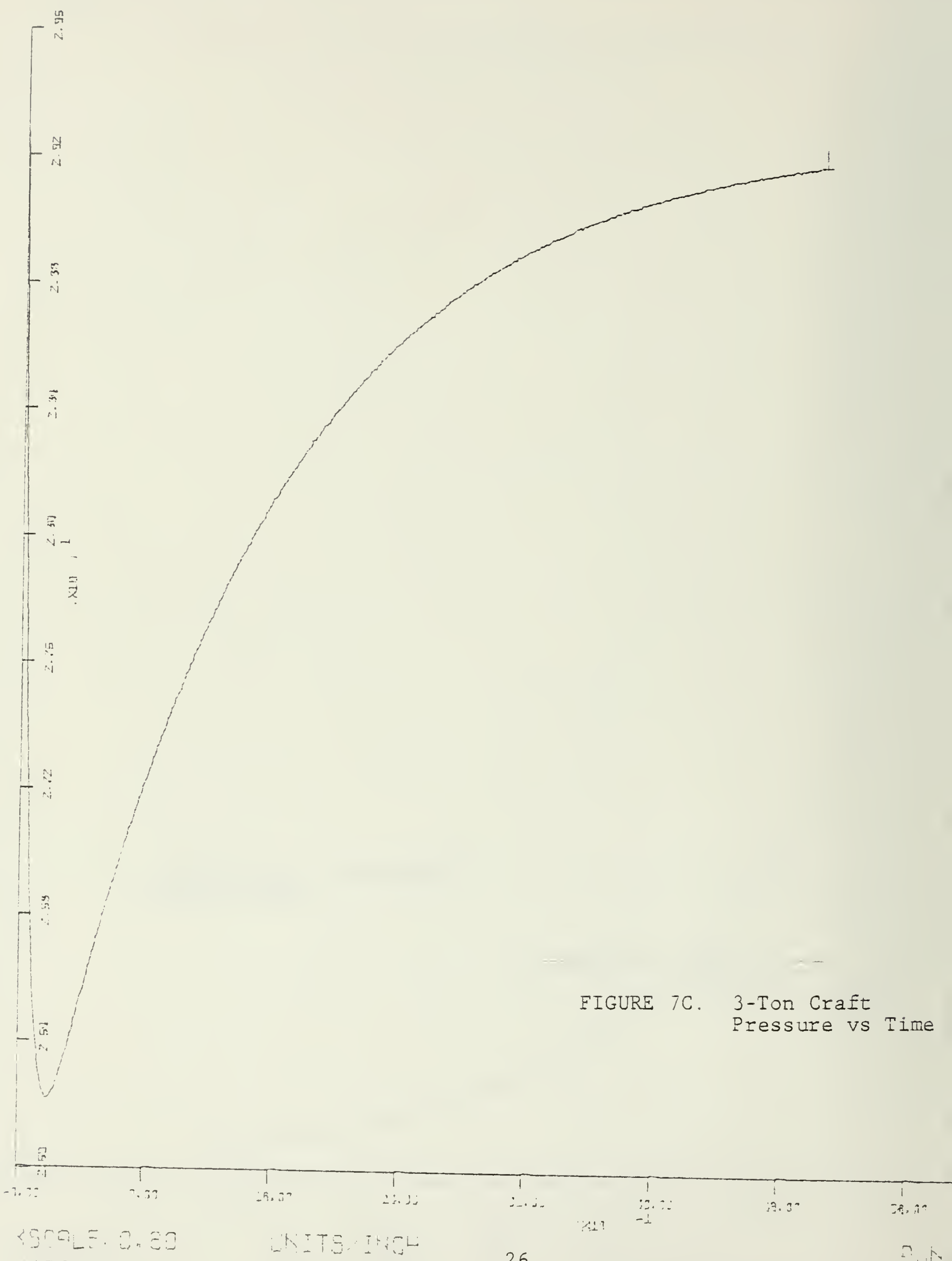


FIGURE 7C. 3-Ton Craft
Pressure vs Time

FIGURE 8A. 3000-Ton Craft
CG Accel vs Time

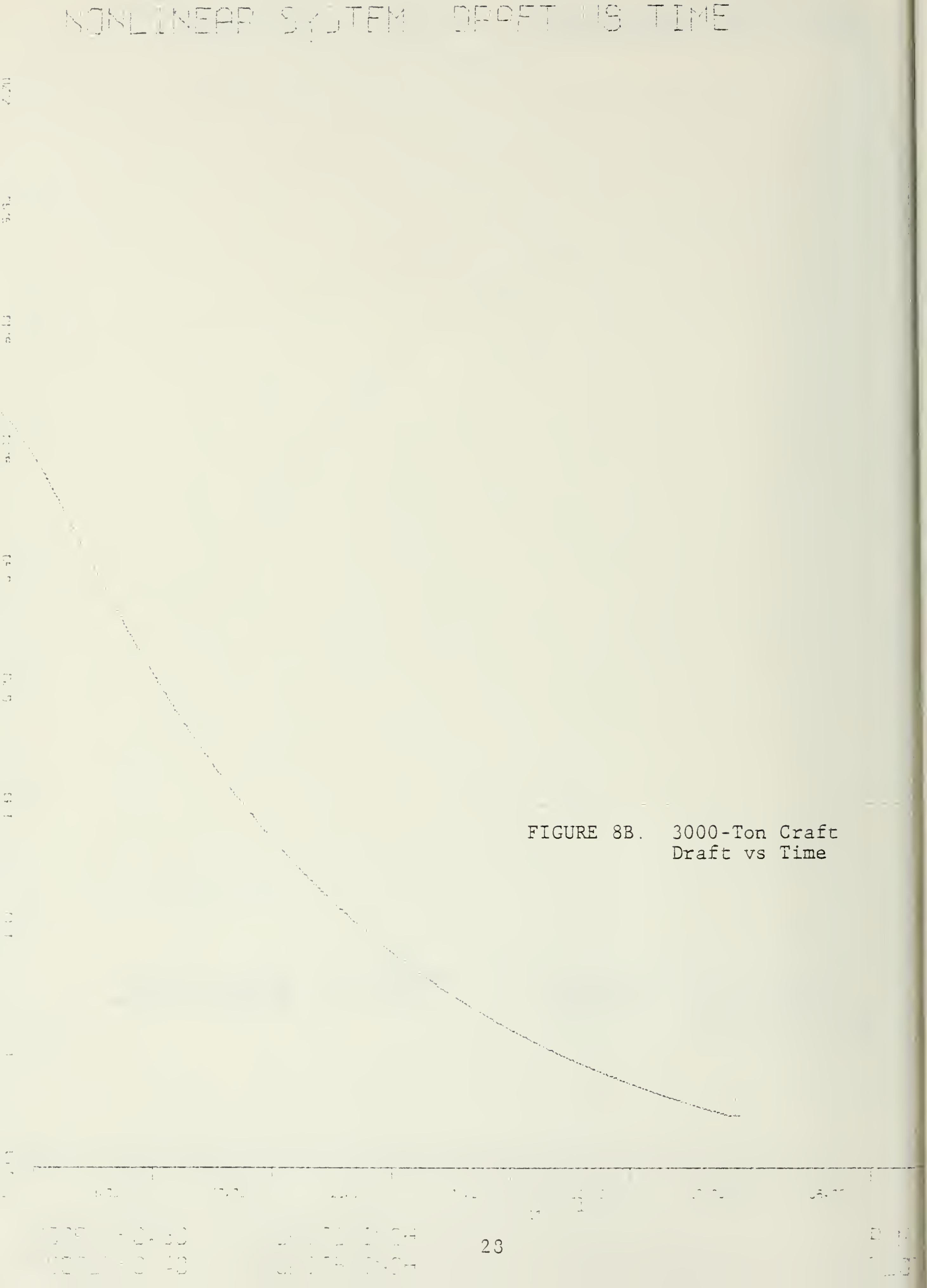


FIGURE 8B. 3000-Ton Craft
Draft vs Time

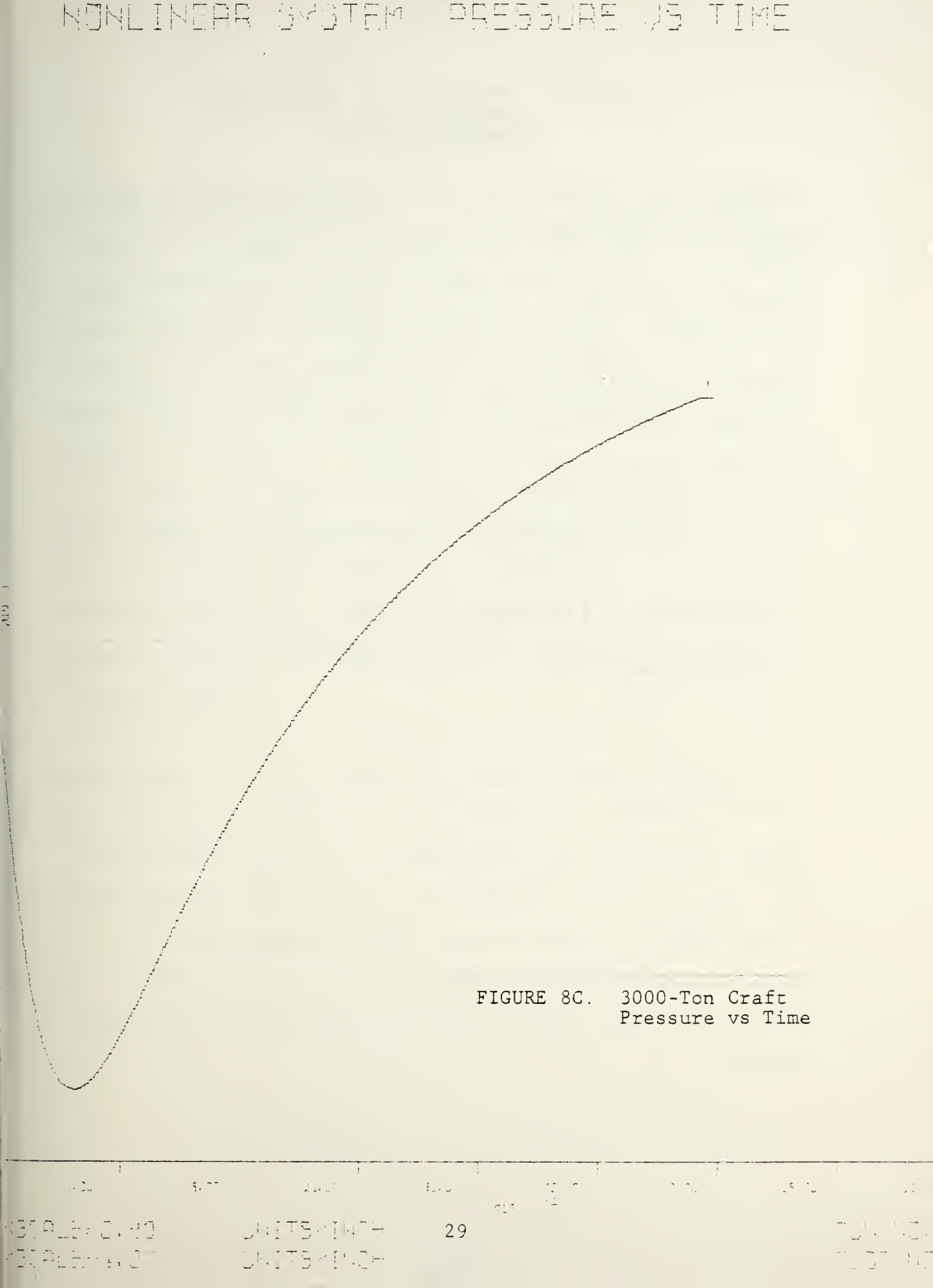


FIGURE 8C. 3000-Ton Craft
Pressure vs Time

TABLE II. Comparison of Scaled Craft Response to a 10 Percent Weight Transient

CRAFT		3-TON	100-TON	3000-TON
Weight (K lbs)		6.72	210	6000
Cushion length (ft)		20.7	65.31	199.3
Scale factor, L_{fs}/L_m		1.0	3.15	9.63
Leakage area (ft^2)		2.5	24.8	231.8
Fan map slope (CFS/PSF)		0.693	6.88	64.27
Plenum pressure (PSF)		29.3	92.2	281.9
Number of fans, n		8	8	8
C.G. Acceleration	t_p, sec	0.425	0.675	1.175
	% overshoot	3.62	6.10	11.79
Draft increment (ft) (0-5) second		0.27	0.82	2.39

It can be observed that the larger craft C.G. acceleration percent overshoot and time of first maximum increase by a facto roughly approximated by the square root of the scale factor, i.e. $\sqrt{\lambda}$. As shown in Reference 1, the vertical motion damping is the result of the pressure effects acting on the C.G. acceleration and does not depend directly on the mass of the craft as it does for the system which has viscous friction damping.

V. CONCLUSIONS

An important consideration in any scaling method used to extend model data to full-sized C.A.B. SES craft is to properly scale leakage area and the fan map characteristic slope. To obtain similar ship characteristics, the same scaling factor used for the linear dimensions should be applied to the leakage area and fan characteristic slope as outlined in Section III.

It was also observed that when scaling up from a three-ton model that the C.G. acceleration percent overshoot and time of the first maximum occur at a value roughly approximate to the square root of the scale factor and do not depend directly upon the mass as it does in systems with viscous friction damping.

VI. RECOMMENDATIONS

The authors feel confident that the procedure outlined in Section III is a valid one for scaling the heave motion equations; however, it remains to be shown that the method will yield similar results for the scaled 6-D.O.F. equations. The task of scaling the 6-D.O.F. program is a lengthy process and was not carried out in this study due to the time constraints. Whenever this task is completed, it would be worthwhile to carry out a frequency response study of the type used in Reference 4 to observe the effect of the scale factor on the C.G. acceleration in order to further validate the procedure used for scaling from model to full-size craft.

Time constraints also prevented the completion of the analysis related to the dependence of the C.G. acceleration characteristics on the square root of the scale factor but the authors feel confident that the characteristic equation of the linear model will yield the desired explanation. We are continuing this study and will include the results obtained in our summary report.

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2. Comstock, J.P. (Editor), Principles of Naval Architecture, Society of Naval Architects and Marine Engineers, New York, 1967.

3. Menzel, R. F., Study of the Roll and Pitch Transients in Calm Water Using the Simulated Performance of the XR-3 Surface Effect Ship Loads and Motions Computer Program, NPS Master Thesis, December 1975.

4. Gerba Jr., A. and Thaler, G. J., Frequency Response Studies of Ambient Pressure Effects on the XR-3 Computer Program (6-D.O.F.), NPS Progress Report to SESPO, September 1977.

APPENDIX A. DSL PROGRAM OF HEAVE MOTION EQ.

```
TITLE HEAVE MOTION DYNAMICS OF A C.A.B. SURFACE EFFECT SHIP
PARAM KL=12.79
INTEGER NPLOT
CONST NPLOT=2
CONTRL FINTIM=25.,DELT=1.25E-04,DELS=.0125
PREPAR .025,TIME,WDOT,VR,LD,VB,MBDOT,MB,PBAR,QIN,QOUT,VM,TD
INITIAL
         KM=0.
         SF=9.63
         DTC=0.001
         PA=2116.
         AS=(75./4.)*(SF**2.)
         RHO=1.99
         G=32.17
         ZS=2.5*SF
         Z=-1.860*SF
         ZIC=-1.860*SF
         ZSS=-2.14*SF
         LDSS=ZSS+ZS
         WT=6720.*(SF**3.)
         M=WT/G
         AB=27 .*(SF**2.)
         VN=383.*(SF**3.)
         GAM=1.4
         RGAM=1./1.4
         RHOA=0.002378
         CN=0.9
         FN=8.
         AL=2.500*(SF**2.)
         LDIC=ZIC+ZS
         FLSS=2.0*AS*RHO*G*LDSS
         FPIC=WT-FLSS
         PBARIC=FPIC/AB
         PBIC=PA+PBARIC
         TDIC=1. *PBARIC
         VBIC=VN-AB*LDIC+KM*TDIC
         PRIC=PBIC/PA
         MBIC=(PRIC**RGAM)*VBIC*RHOA
         QOUTIC=CN*AL*SQRT(2.0*PBARIC/RHOA)
         KQ=.593*(SF**2.0)
         QIC=(QOUTIC/FN)+PBARIC*KQ
         WIC=0.0
         PBAR=PBARIC
         TD=PBARIC
         PBAR=29.3
         TD=24.0
DERIVATIVE
         LD=Z+ZS
         FLM=(2.0*AS*RHO*G*LD)/M
         VM=MB/RHOA
         VB=VN-AB*LD+KM*PBAR
         AVR=ABS(VM/VB)
         VR=(AVR)**GAM*SIGN(1.,VM)*SIGN(1.,VB)
         PB=VR*PA
         PBAR=PB-PA
         FPM=AB*PBAR/M
         WDOT=WT/M-FPM-FLM
         W=INTGRL(WIC,WDOT)
         QIN=FN*(QIC-PBAR*KQ)
         APBAR=ABS(PBAR)
         QOUT=CN*AL*SQRT(2.*APBAR/RHOA)*SIGN(1.,PBAR)
         FLOW=QIN-QOUT
         MBDOT=RHOA*FLOW
```

```
      MB=INTGRL(MBIC,MBDOT)
      Z=INTGRL(ZIC,W)
SAMPLE
PRINT 0.25,TIME,WDOT,VR,LD,VB,MBDOT,MB,PBAR,QIN,QOUT,VM
PRPLOT
GRAPH SAME,TIME,LD
GRAPH SAME,TIME,PBAR
GRAPH SAME,TIME,VR
GRAPH SAME,TIME,WDOT
GRAPH SAME,TIME,VB,VM
GRAPH SAME,TIME,TD
      CALL DRWG(1,1,TIME,LD)
      CALL DRWG(2,1,TIME,PBAR)
      CALL DRWG(3,1,VB,VM)
      CALL DRWG(4,1,TIME,VR)
      CALL DRWG(5,1,TIME,WDOT)
TERMINAL
      CALL ENDRW(NPLOT)
      KL=KL+100.
      IF(KL.GT.130.) CALL ENDJOB
      CALL RERUN
END
STOP
//PLOT.SYSIN DD *
N29 BOX 70 GERBA   PLENUM PRESSURE DYNAMICS
    NONLINEAR SYSTEM  DRAFT VS TIME
N29 BOX 70 GERBA   PLENUM PRESSURE DYNAMICS
     NONLINEAR SYSTEM  PRESSURE VS TIME
N29 BOX 70 GERBA   PLENUM PRESSURE DYNAMICS
  NL SYSTEM AIR VOLUME VS PLENUM VOLUME
N29 BOX 70 GERBA   PLENUM PRESSURE DYNAMICS
    NONLINEAR SYSTEM PRESSURE RATIO VS TIME
N29 BOX 70 GERBA   PLENUM PRESSURE DYNAMICS
    NONLINEAR SYSTEM CG ACCEL VS TIME
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APPENDIX B. LINEAR SYSTEM EQUATIONS

The linear system equations are arrived at by application of the Taylor Series expansion[a] about the steady-state operating point. The development is shown below for the two differential equations.

Equation (1), let $W/M = W_m$; $F_p/M = F_{pm}$

and $F_\ell/M = F_{\ell m}$

then from equation 6 in Section II,

$$\ddot{z}(o) + \delta\ddot{z} = W/M - (F_{pm}(o) + \delta F_{pm}) - (F_{\ell m}(o) + \delta F_{\ell m})$$

or,

$$\boxed{\delta\ddot{z} = -\delta F_{pm} - \delta F_{\ell m}} \qquad (1a)$$

Equation (2), let $m/\rho_a = V_m$ for convenience

then, from equation 5 in Section II,

$$\dot{V}_m(o) + \delta\dot{V}_m = nQ_i - nk_q\ (\overline{P}_b(0) + \delta\overline{P}_b)$$

$$- C_n A_\ell \left(\frac{2\overline{P}_b(o)}{\rho_a}\right)^{\frac{1}{2}}$$

$$- C_n A_\ell\ \tfrac{1}{2}\left(\frac{2\overline{P}_b(o)}{\rho_a}\right)^{-\frac{1}{2}} \left(\frac{2}{\rho_a}\right)\ \delta\overline{P}_b$$

or,

$$\boxed{\delta\dot{V}_m = -(nk_q)\delta\overline{P}_b - \frac{C_n A_\ell}{\rho_a}\left(\frac{\rho_a}{2\overline{P}_b(o)}\right)^{\frac{1}{2}} \delta\overline{P}_b} \qquad (2a)$$

(a) Steady state and incremental variations are noted as follow using heave as an example.

$$z = z(o) + \delta z$$

Next, the auxiliary algebraic equations are linearized.

$$F_p(o) + \delta F_p = A_b\ (\overline{P}_b\ (o) + \delta\overline{P}_b)$$

$$\boxed{\delta F_p = A_b\ \delta\overline{P}_b} \qquad (3a)$$

$$F_\ell(o) + \delta F_\ell = 2A_s\rho_w g\ (\ell_d(o) + \delta\ell_d)$$

$$\boxed{\delta F_\ell = (2A_s\rho_w g)\ \delta\ell_d} \qquad (4a)$$

Following the above procedure, equations 1 and 2 from Section II are linearized with the results given in equations (5a) and (6a).

$$\boxed{\delta q_{in} = -(nk_q)\ \delta\overline{P}_b} \qquad (5a)$$

and

$$\boxed{\delta q_{out} = -\frac{C_n A_\ell}{\rho_a}\left(\frac{\rho_a}{2\overline{P}_b(o)}\right)^{\frac{1}{2}}\delta\overline{P}_b} \qquad (6a)$$

For equation 3 in the report, the equation becomes,

$$\frac{(P_b(o) + \delta P_b)}{P_a} = \left(\frac{V_m(o)}{V_b(o)}\right)^{\gamma} + \gamma\left(\frac{V_m(o)}{V_b(o)}\right)^{\gamma-1}$$

$$\left\{\left[\frac{1}{V_b(o)}\right]\delta V_m - \frac{(V_m(o))}{[V_b(o)]^2}\delta V_b\right\}$$

$$\delta P_b = P_a\gamma\ (\quad)^{\gamma-1}\left\{\left[\frac{V_m(o)}{V_b(o)}\ \frac{\delta V_m}{V_m(o)} - \frac{V_m(o)}{V_b(o)}\ \frac{\delta V_b}{V_b(o)}\right.\right\}$$

$$\boxed{\delta P_b = \gamma P_b(o)\left[\frac{1}{V_m(o)}\ \delta V_m - \frac{1}{V_b(o)}\ \delta V_b\right]} \qquad (7a)$$

and linearized equations 4, 11 and 12 follow.

$$V_b(o) + \delta V_b = V_n - A_b\ (\ell_d(o) + \delta\ell_d)$$

$$\boxed{\delta V_b = -\ (A_b)\,\delta\ell_d \qquad\qquad (8a)}$$

$$\bar{P}_b(o) + \delta\bar{P}_b = P_b(o) + \delta P_b - P_a$$

$$\boxed{\delta\bar{P}_b = \delta P_b \qquad\qquad (9a)}$$

$$\ell_d(o) + \delta\ell_d = z(o) + \delta z + Z_s$$

$$\boxed{\delta\ell_d = \delta z \qquad\qquad (10a)}$$

Combining (1a), (3a), (4a), (7a) and (8a) as follows:

$$\delta\ddot{z} = -\left(\frac{A_b}{M}\right)\delta\bar{P}_b - \left(\frac{2A_s\rho_w g}{M}\right)\delta z$$

recall that $\delta\ell_d = \delta z$

$$\delta\ddot{z} = -\frac{A_b}{M}\left\{\gamma\ P_b(o)\left[\frac{1}{V_m(o)}\,\delta V_m - \frac{1}{V_b(o)}\,(-A_b)\ \delta z\right]\right\}$$

$$-\left(2\ \frac{A_s\rho_w g}{M}\right)\ \delta z$$

$$\delta\ddot{z} = -\left\{\left[\left(\frac{2A_s\rho_w g}{M}\right) + \left(\frac{\gamma P_b(o)A_b^2}{MV_b(o)}\right)\right]\delta z + \left[\frac{\gamma P_b(o)A_b}{MV_m(o)}\right]\delta V_m\right\}$$

Also for equation (2a)

$$\delta\dot{V}_m = -\left[nk_q + \frac{C_n A_\ell}{\rho_a}\left(\frac{\rho_a}{2\bar{P}_b(o)}\right)^{\frac{1}{2}}\right] \gamma P_b(o) \left[\frac{\delta V_m}{V_m(o)} - \frac{\delta V_b}{V_b(o)}\right]$$

$$\delta\dot{V}_m = -\left[nk_q + \frac{C_n A_\ell}{\rho_a}\left(\frac{\rho_a}{2\bar{P}_b(o)}\right)^{\frac{1}{2}}\right] \gamma P_b(o) \left[\left(\frac{1}{V_m(o)}\right)\delta V_m\right.$$

$$\left. + \left(\frac{A_b}{V_b(o)}\right) \delta z\right]$$

and for equation (7a)

$$\delta P_b = \gamma P_b(o) \left[\frac{1}{V_m(o)} \delta V_m + \frac{A_b}{V_b(o)} \delta z\right]$$

Now define the state vector

$$\underline{x} \triangleq [\delta z \quad \delta\dot{z} \quad \delta V_m]^T$$

and output variable, $y = \delta P_b$ with

$$y \triangleq c_{11}\ x_1 + c_{13}\ x_3$$

Then the state equations are

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = a_{21}\ x_1 + a_{23}\ x_3$$

$$\dot{x}_3 = a_{31}\ x_1 + a_{33}\ x_3$$

<u>where</u> $c_{11} = \dfrac{\gamma P_b(o) A_b}{V_b(o)}$ and $c_{13} = \dfrac{\gamma P_b(o)}{V_m(o)}$

$$a_{21} = -\left(\frac{2A_s \rho_w g}{M} + \frac{\gamma P_b(o) A_b^{\ 2}}{MV_b(o)}\right)$$

$$a_{23} = -\left(\frac{\gamma P_b(o) A_b}{MV_m(o)}\right)$$

$$a_{31} = -\gamma P_b(o)\left[nk_q + \frac{C_n A_\ell}{\rho_a}\sqrt{\frac{\rho_a}{2\bar{P}_b(o)}}\right]\frac{A_b}{V_b(o)}$$

$$a_{33} = -\gamma P_b(o)\left[nk_q + \frac{C_n A_\ell}{\rho_a}\sqrt{\frac{\rho_a}{2\bar{P}_b(0)}}\right]\frac{1}{V_m(o)}$$

The characteristic equation for this system is given by $(sI - A)$ where

$$A = \begin{vmatrix} o & 1 & o \\ a_{21} & o & a_{23} \\ a_{31} & o & a_{33} \end{vmatrix}$$

Then $(sI - A) = \begin{bmatrix} s & -1 & o \\ -a_{21} & s & -a_{23} \\ -a_{31} & o & s - a_{33} \end{bmatrix}$

and the characteristic equation is

$$s^3 - a_{33}\, s^2 - a_{21}\, s + (a_{21}\, a_{33} - a_{23}\, a_{31}) = o$$

Using the numerical values given in Reference 1, the characteristic equation is

$$s^3 + 62.s^2 + 2133s + 789 = o$$

This cubic equation will factor into one real root near the origin and a complex pair far removed from the jw axis. Thus, a step weight response will have a complex pair of poles which can be approximated by the quadratic form,

$$s^2 - a_{33}s - a_{21} = o$$

For the given numerical values that approximate the XR-3 craft, it can be shown that a rough approximation for the coefficients are:

$$-a_{33} \doteq nk_q \, \gamma P_b(o)/V_m(o)$$

$$-a_{21} \doteq \frac{A_b^{\,2}}{M} \, \gamma \, P_b(o)/V_b(o)$$

Using the above approximation yields a natural frequency of:

$$w_n = \sqrt{-a_{21}} = \left[\frac{A_b^{\,2}}{M} \quad \gamma \quad P_b(o)/V_b(o) \right]^{\frac{1}{2}}$$

$$w_n = K_n \sqrt{P_b(o)/V_b(o)} \tag{13}$$

where $K_n = A_b \sqrt{\gamma/M}$

and a damping factor

$$\zeta = \frac{-a_{33}}{2w_n} = \left[\frac{n^2 k_q^{\,2} \quad \gamma^2 \quad P_b^{\,2}(o) \;/\; V^2{}_m(o)}{4 \quad \frac{A_b^{\,2}}{M} \; \gamma \; P_b(o) \;/\; V_b(o)} \right]^{\frac{1}{2}}$$

$$\zeta = K_f \sqrt{\frac{P_b(o) \quad V_b(o)}{V^2{}_m(o)}} \tag{14}$$

where

$$K_f = \frac{n \; k_q}{2A_b} \sqrt{\gamma \; M}$$

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